Measuring small subgroup attacks against Diffie-Hellman

Luke Valenta[*], David Adrian[†], Antonio Sanso[‡], Shaanan Cohney[*], Joshua Fried[*], Marcella Hastings[*], J. Alex Halderman[†], Nadia Heninger[*]

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February 28, 2017

This work

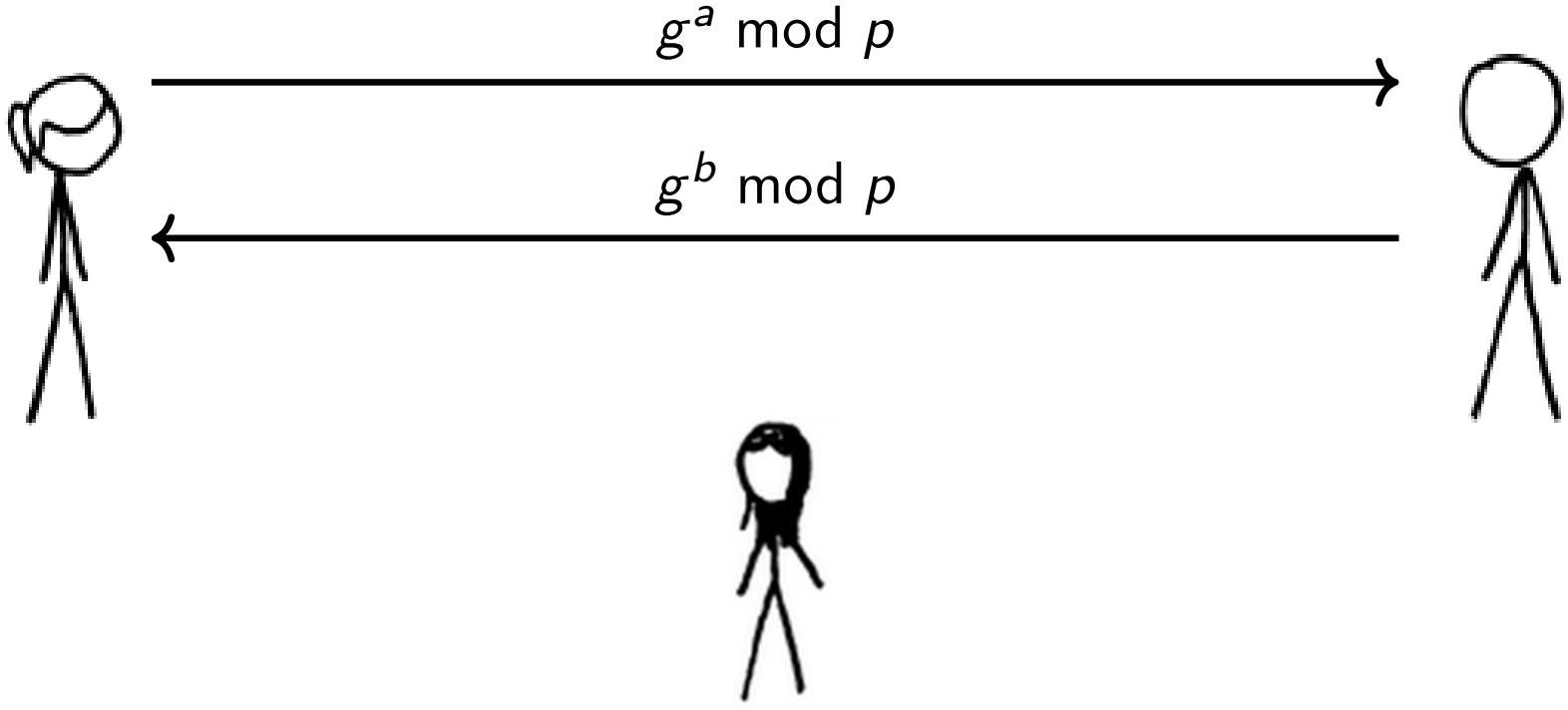
- Revisit decades-old small subgroup attacks in Diffie-Hellman
- Looked at hosts and implementations in the wild
- Punch line: Nobody implements the countermeasures!
- Emerged from **Logjam** [ABDGGHHSTVVWZZ 2015]

Textbook (Finite-Field) Diffie-Hellman Key Exchange

[Diffie Hellman 1976]

p a prime (so $\mathbb{F}_p^*$ is a cyclic group)

$g < p$ group generator (often 2 or 5)



Images from XKCD

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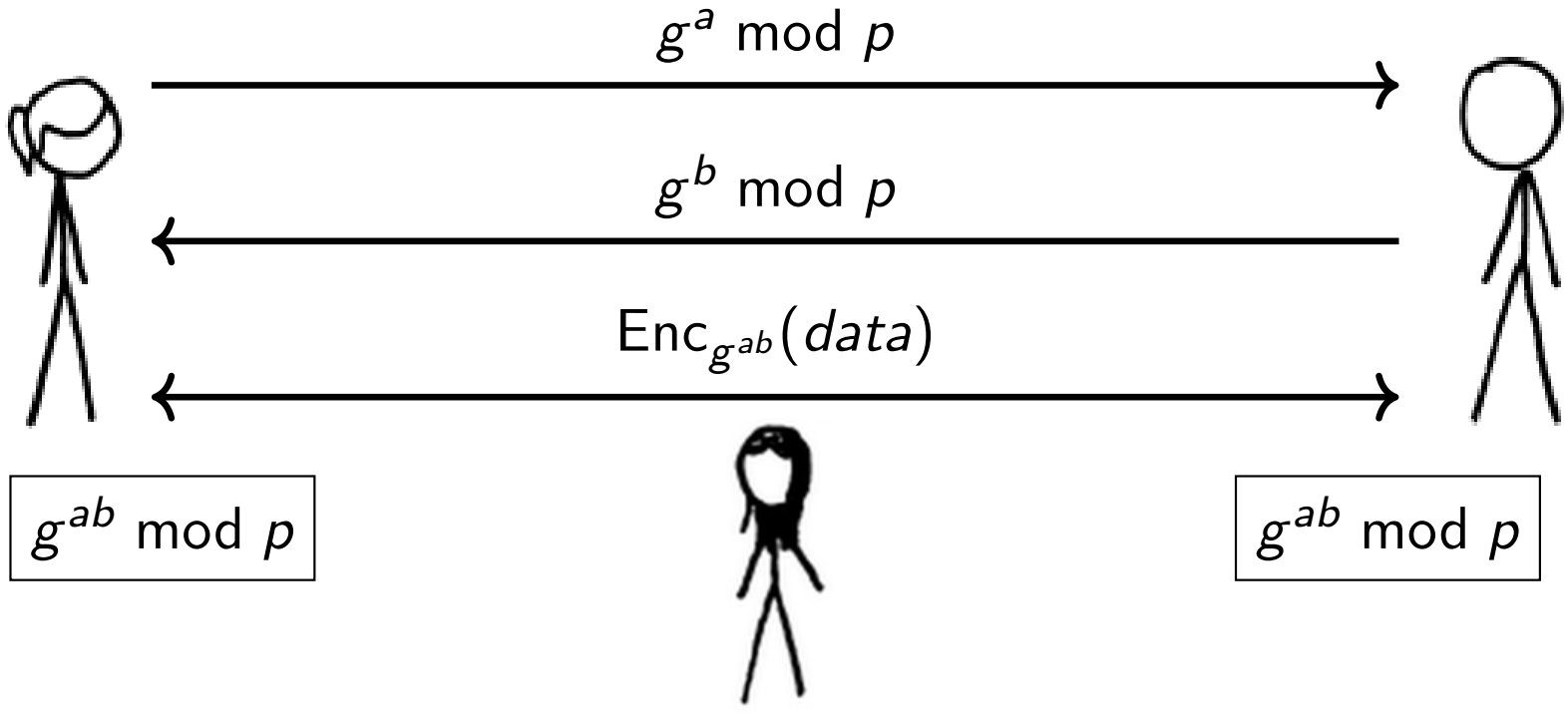

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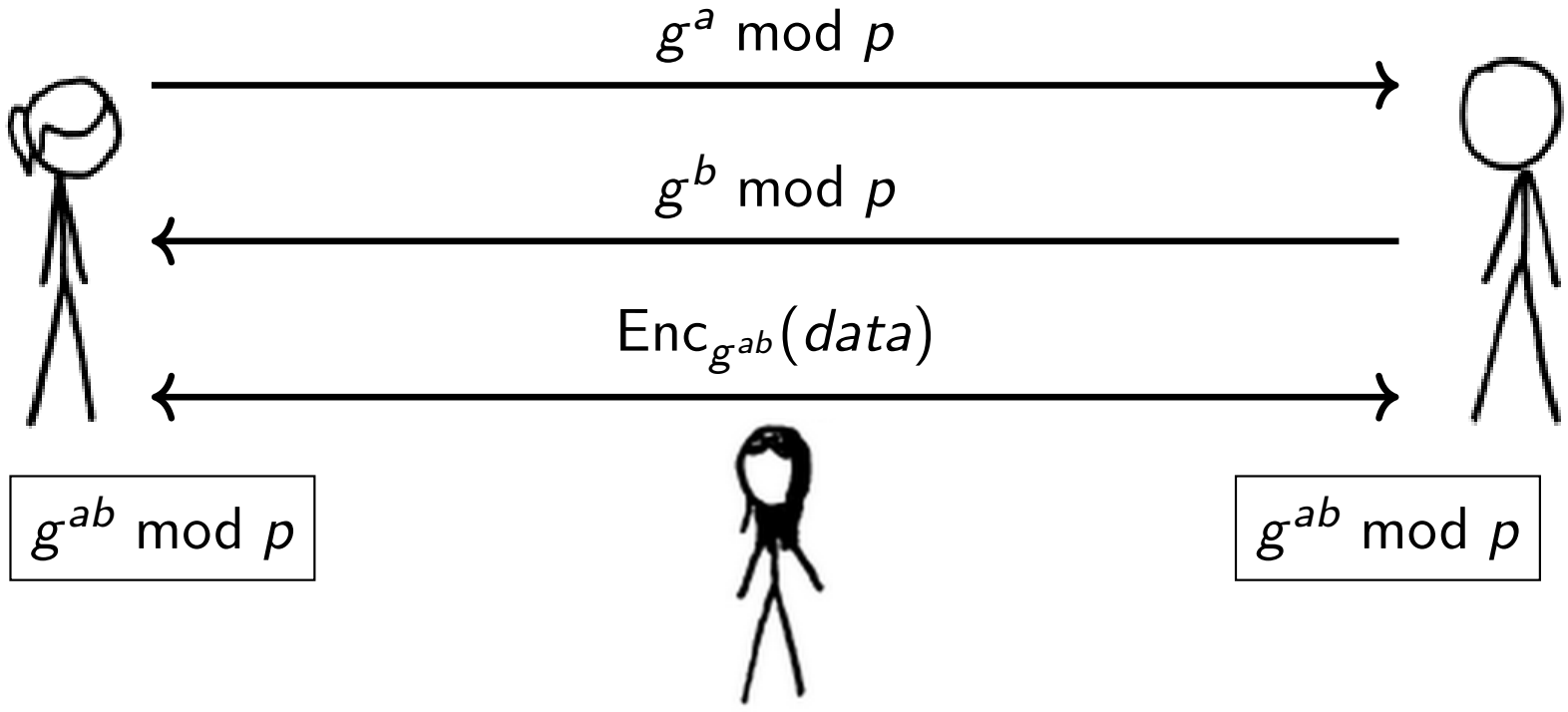

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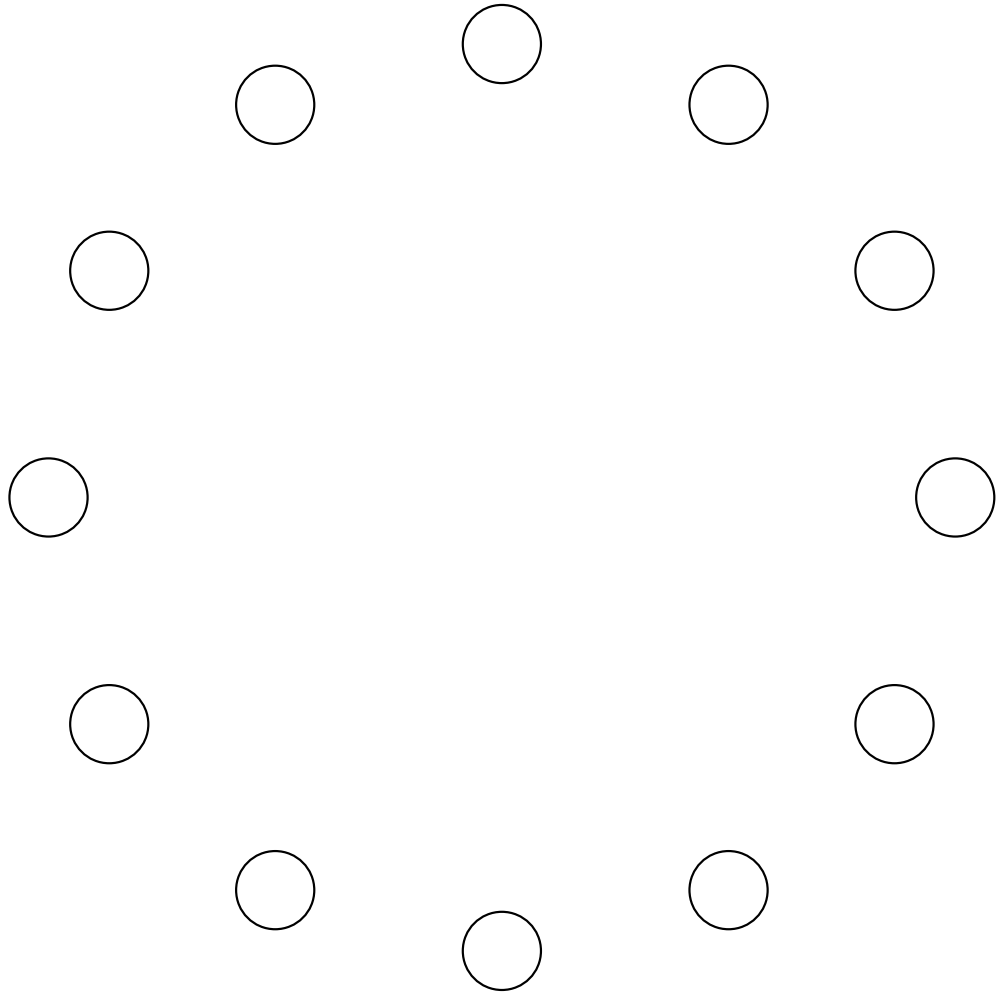
NH: "There are dragons swimming under the placid surface of this beautiful mathematical lake."

Background: groups, subgroups, and generators

Cyclic group

Order = #elements in group



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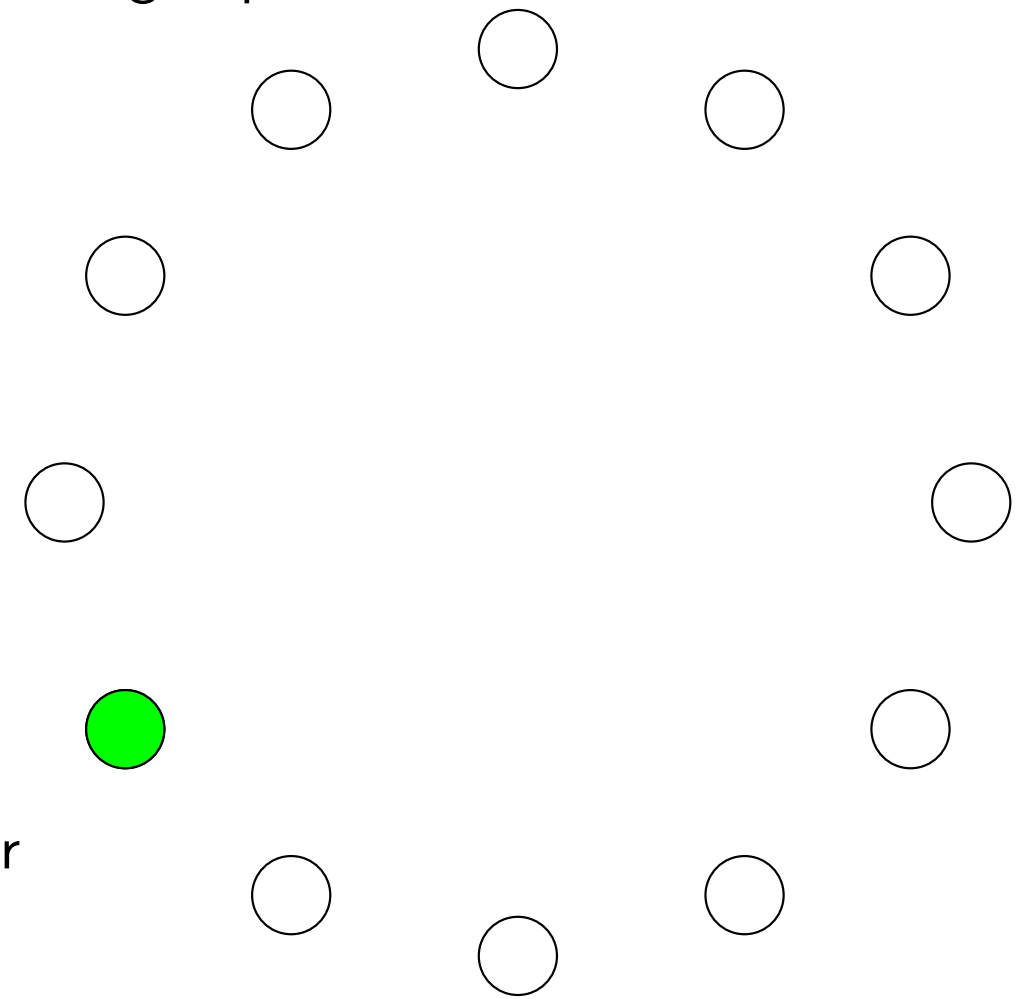

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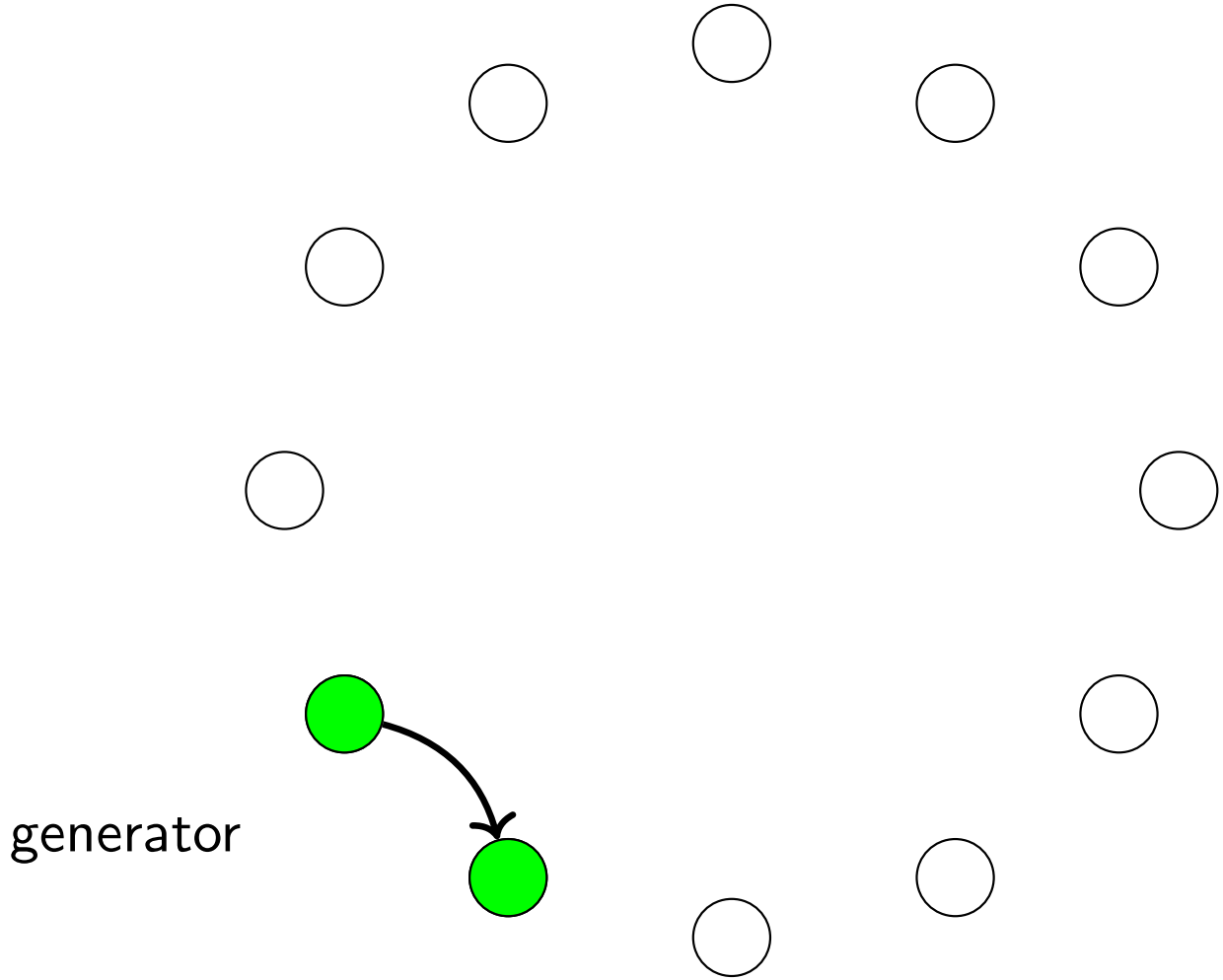

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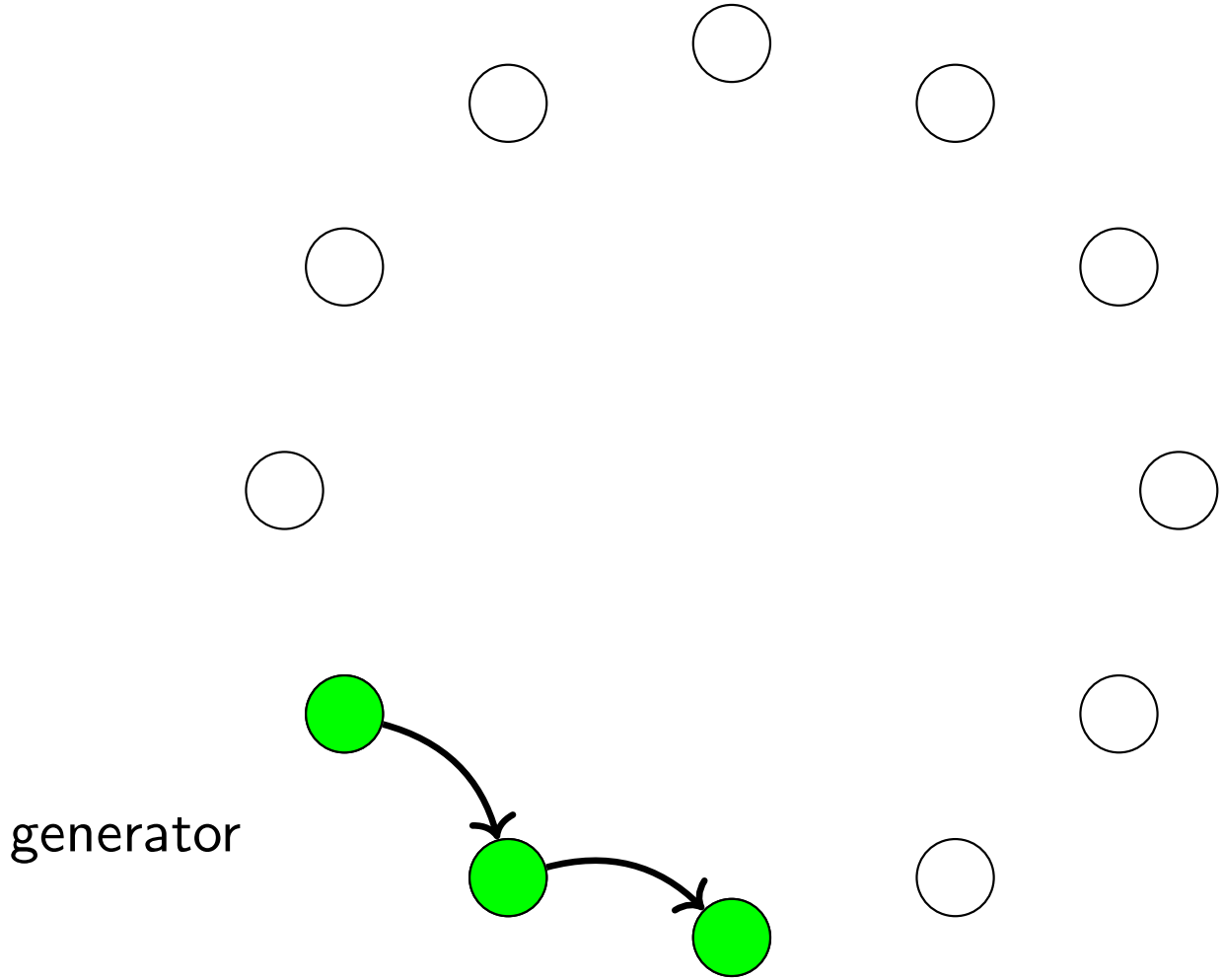

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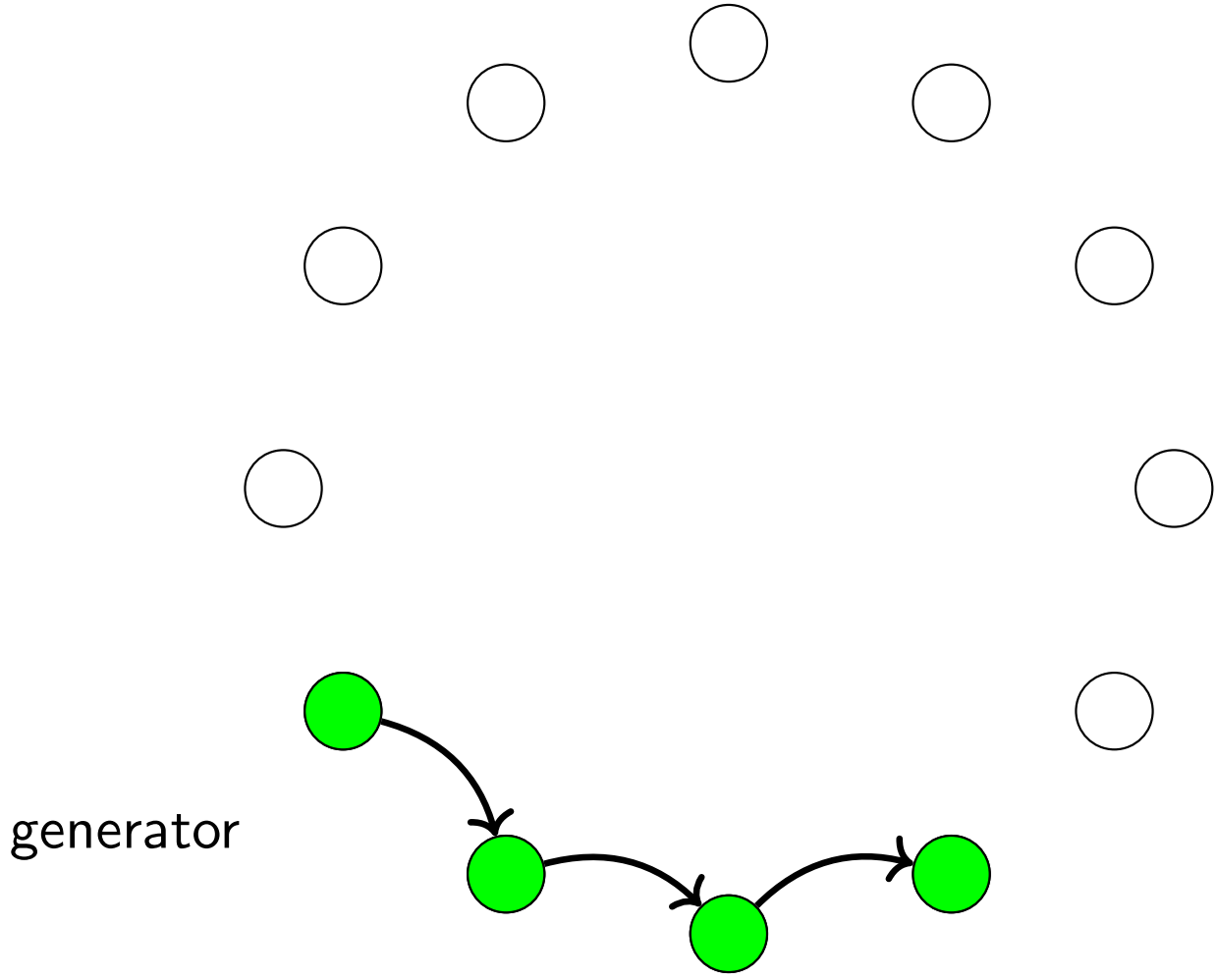

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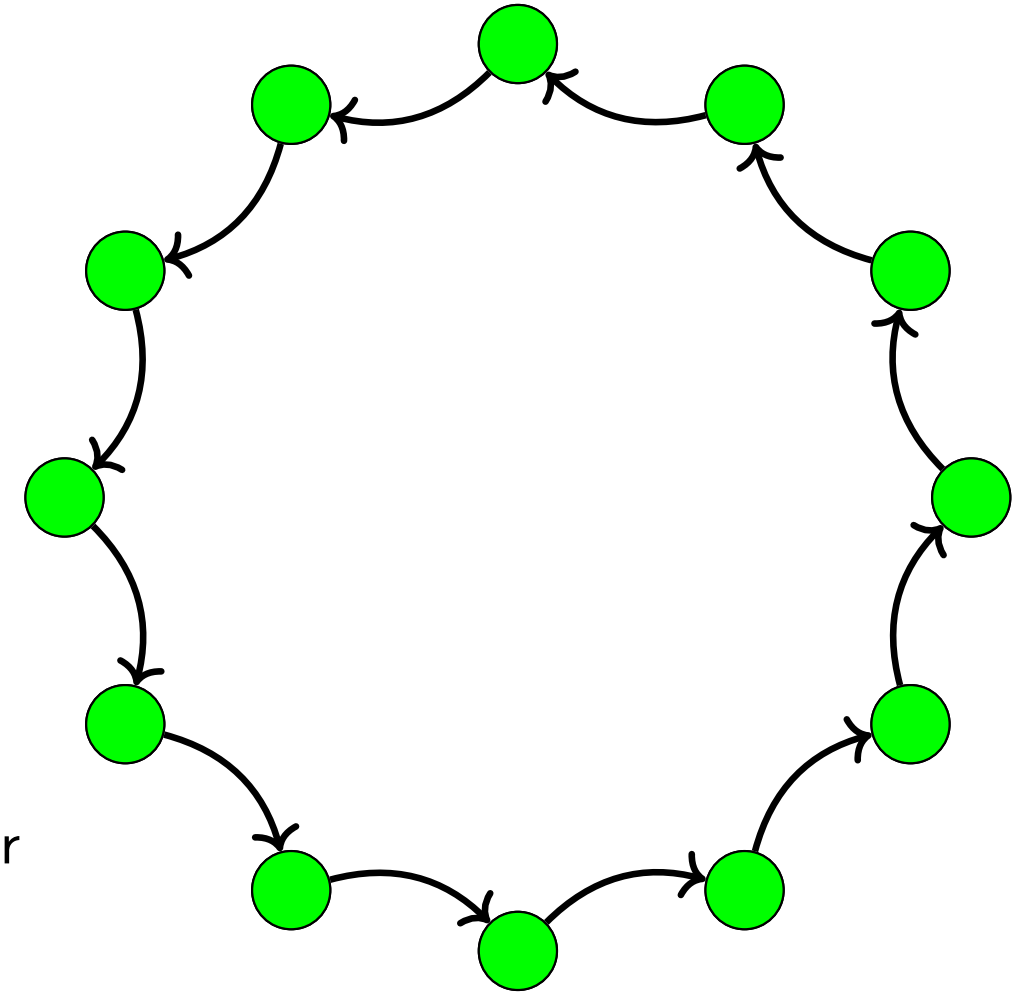

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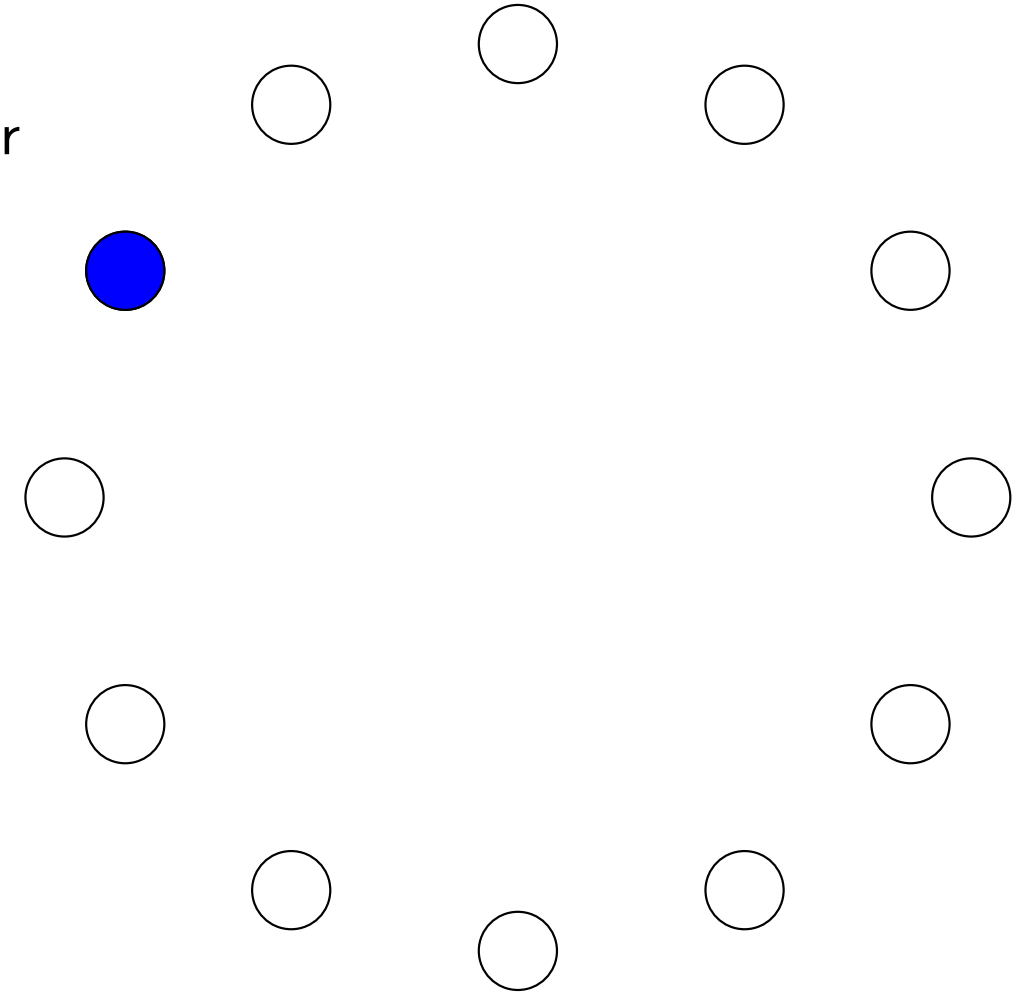

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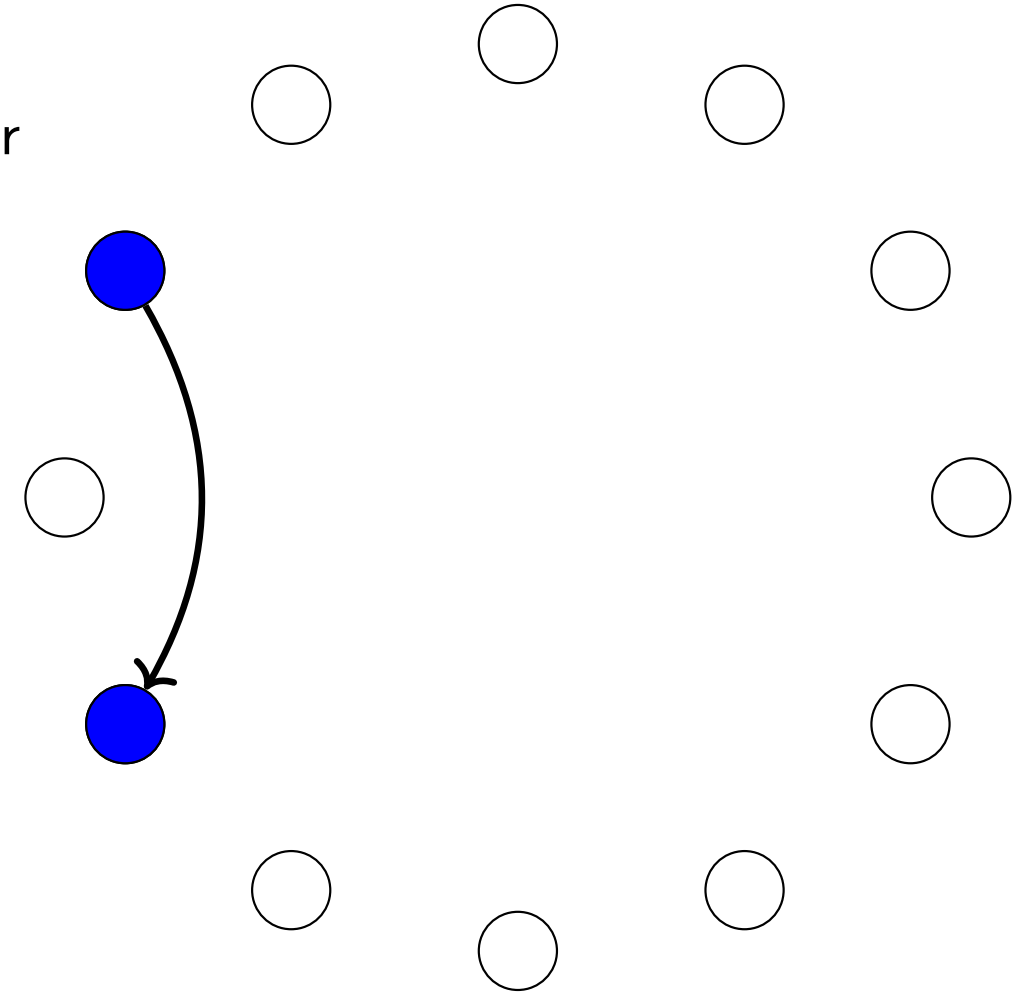

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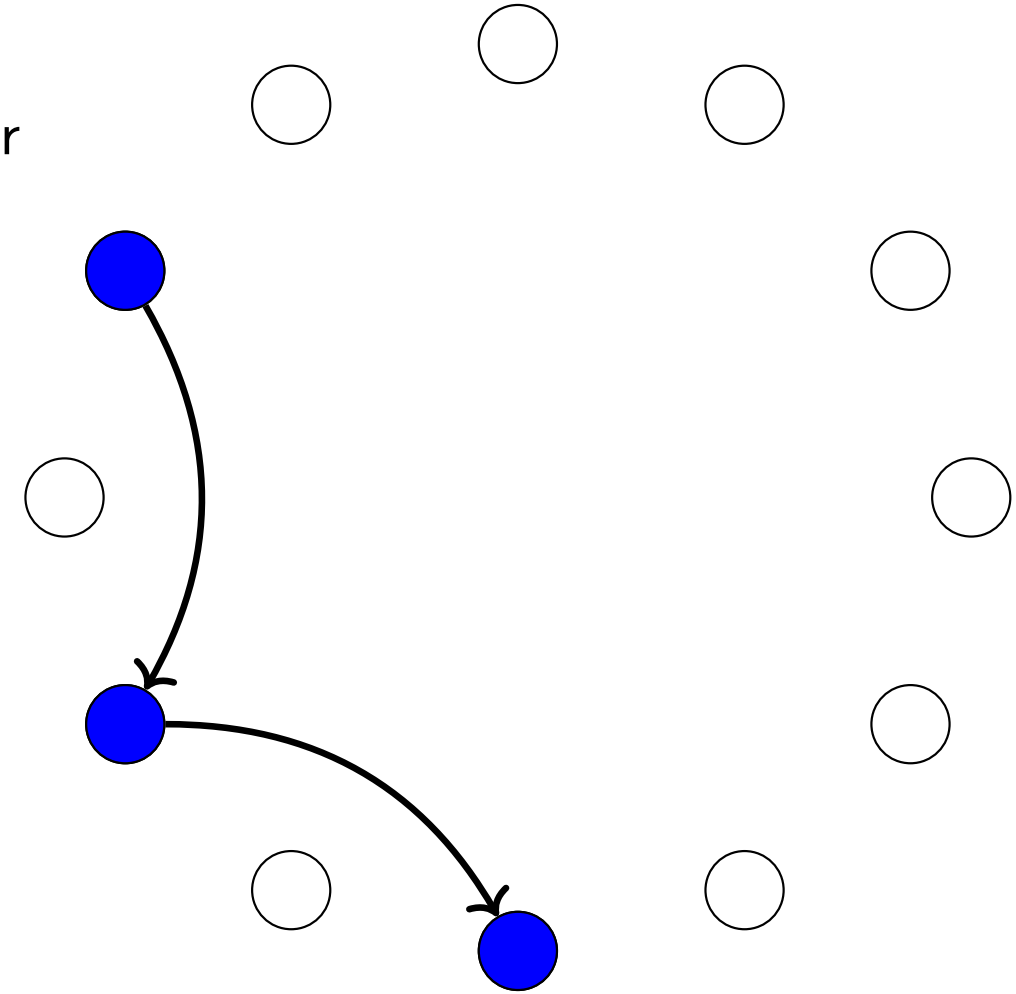

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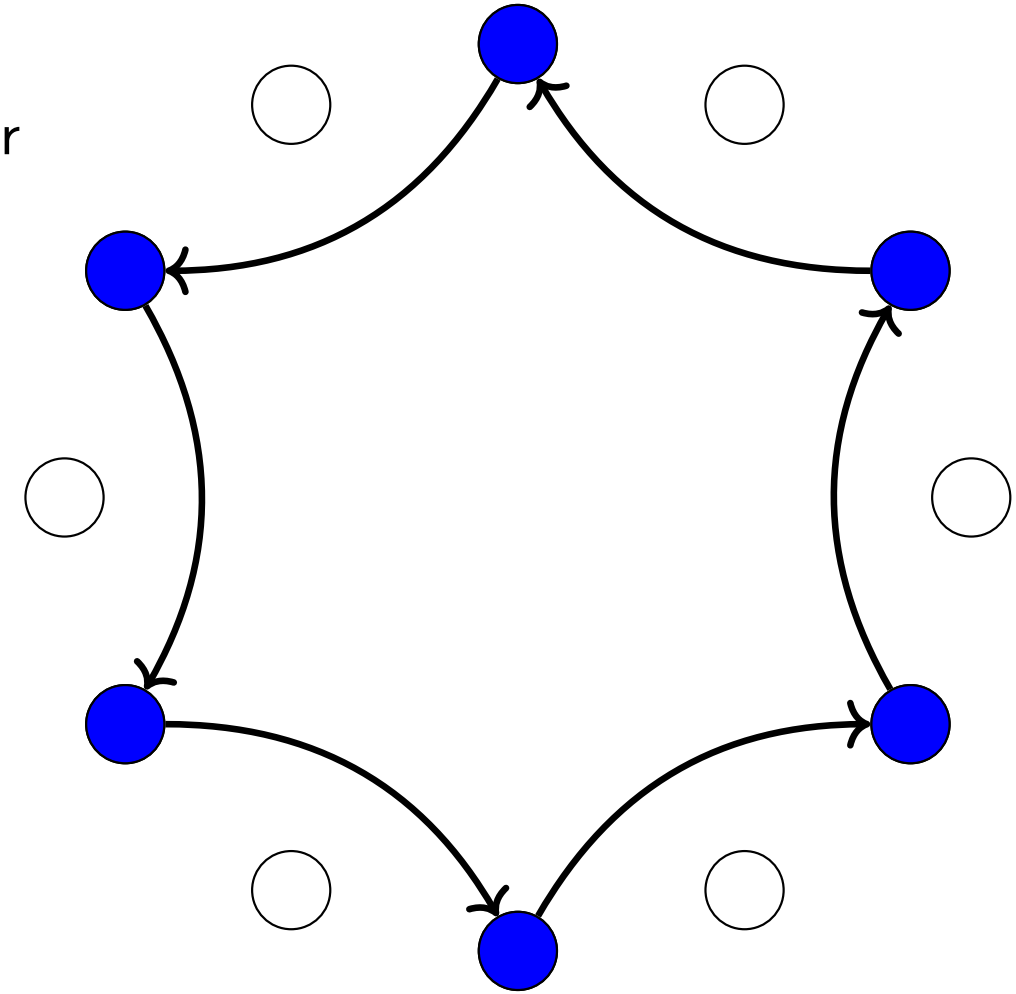

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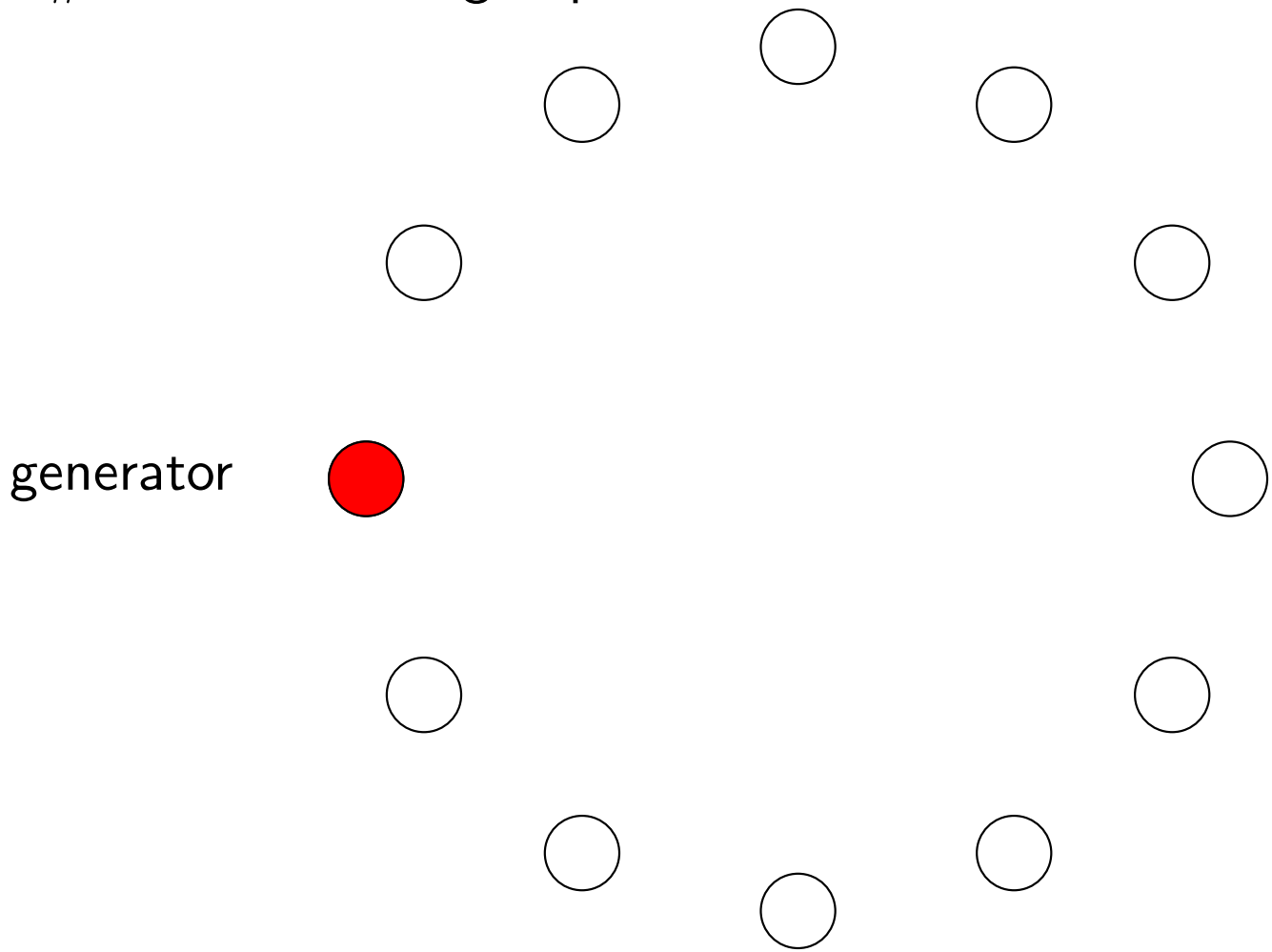

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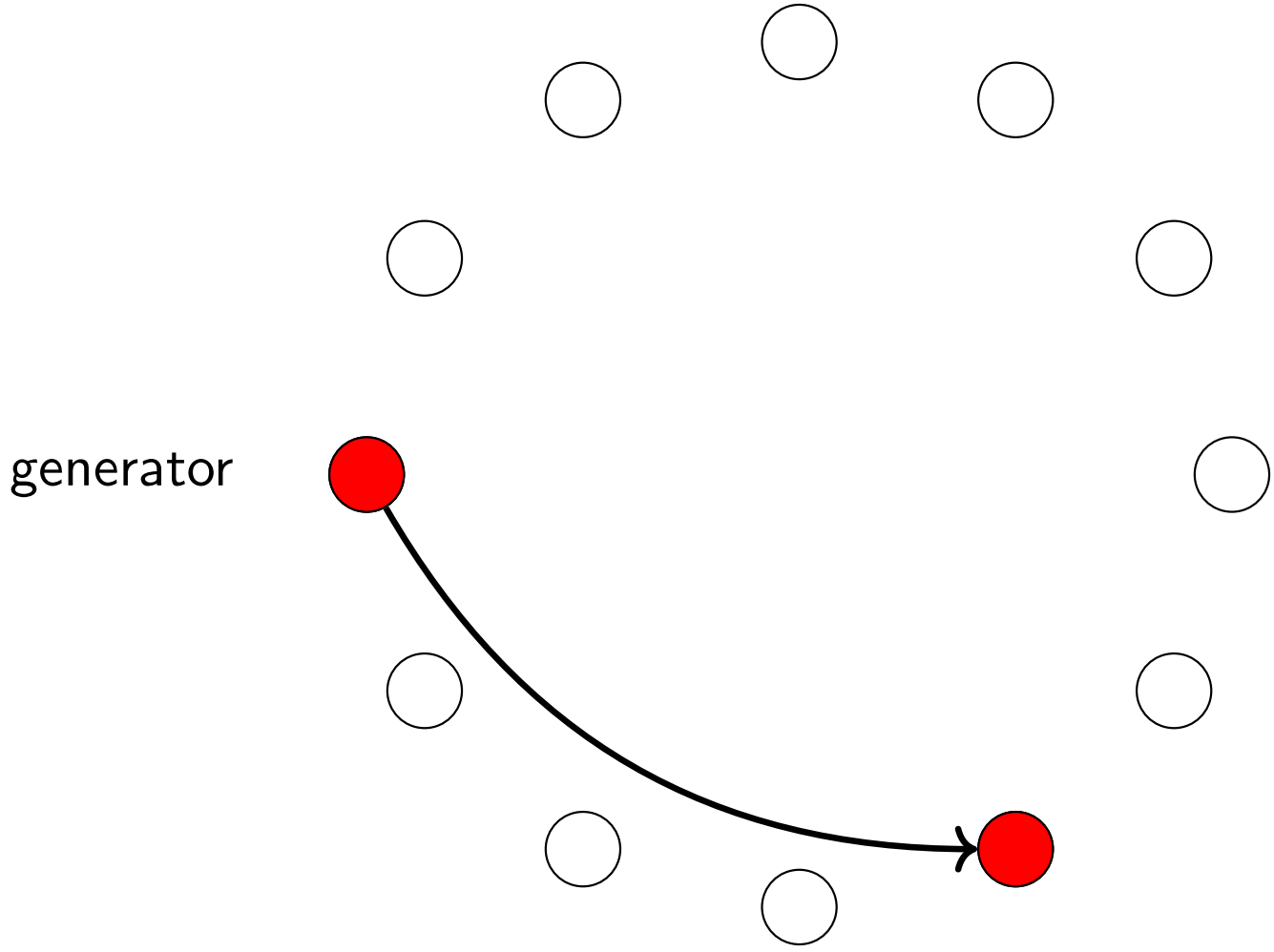

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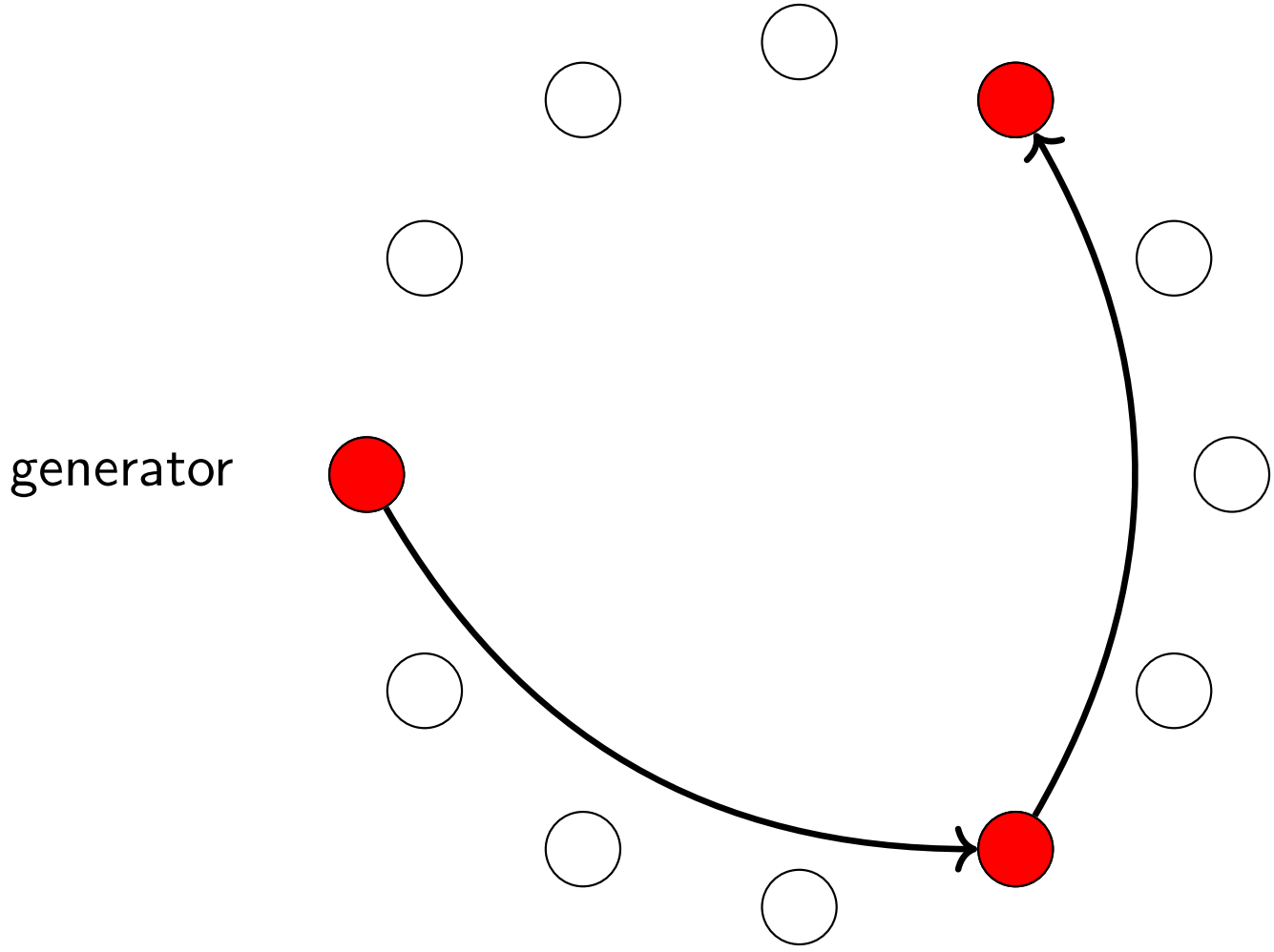

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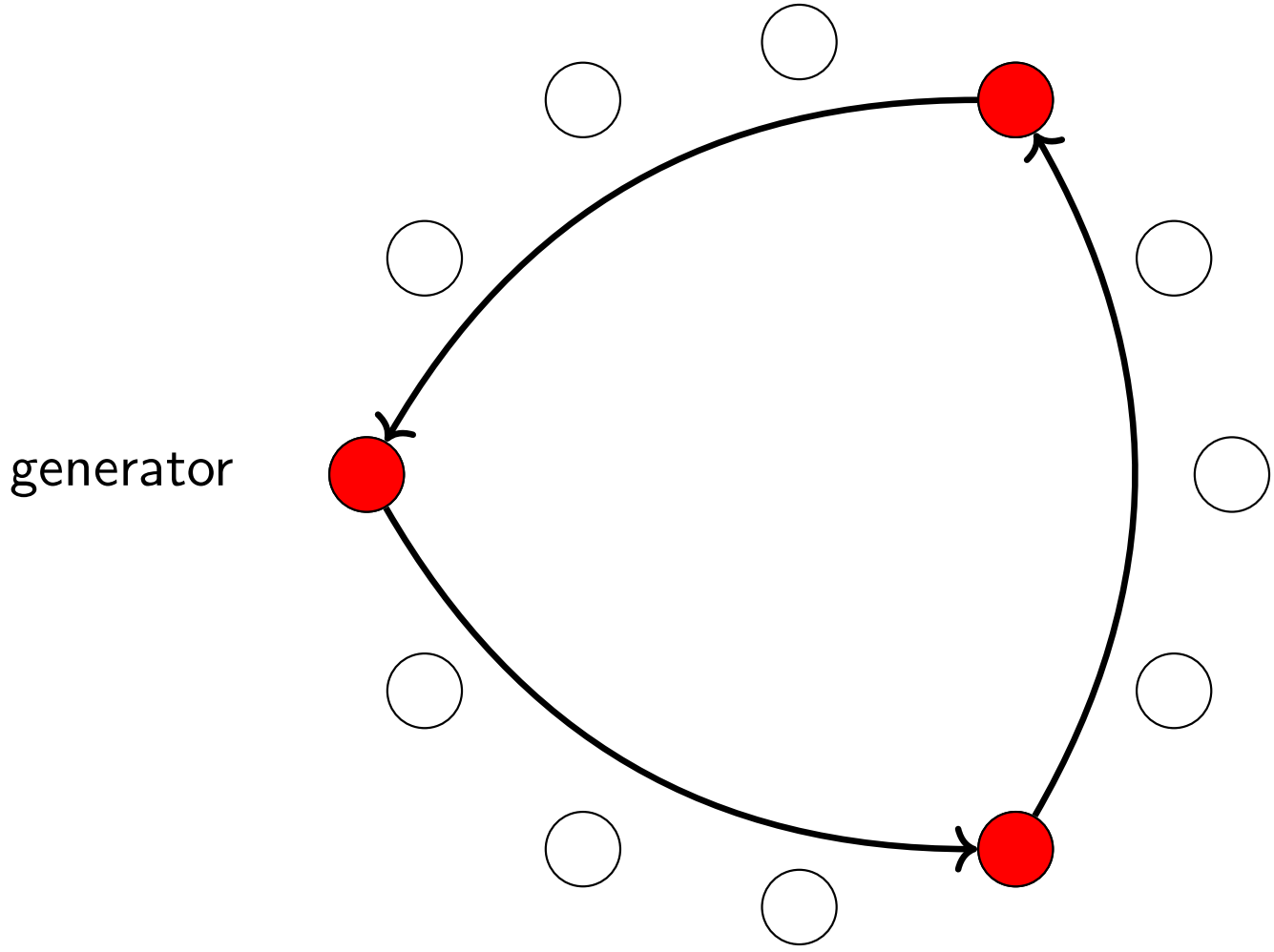
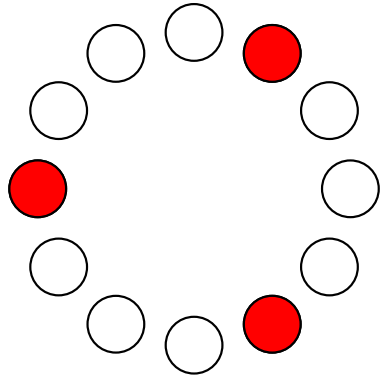


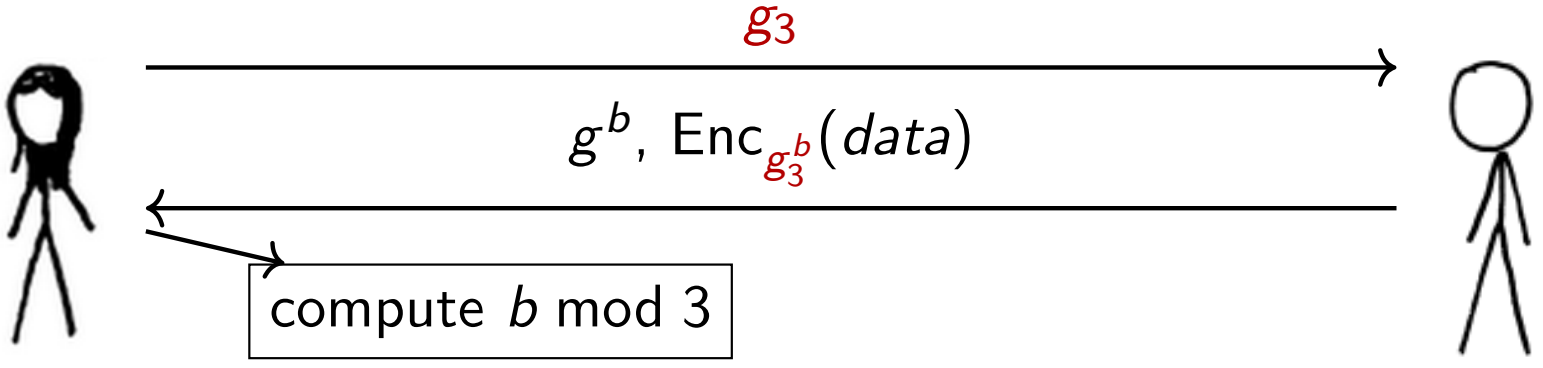
Existence of small subgroups $\rightarrow$ small subgroup attacks.

g generates correct subgroup of order q

g_3 generates subgroup of order 3



[Lim Lee 1997]



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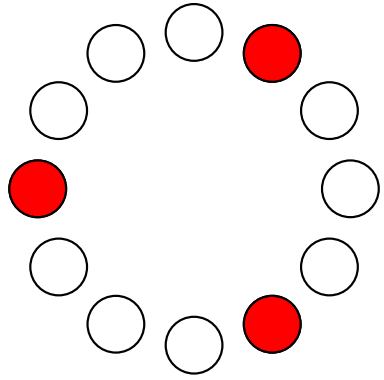

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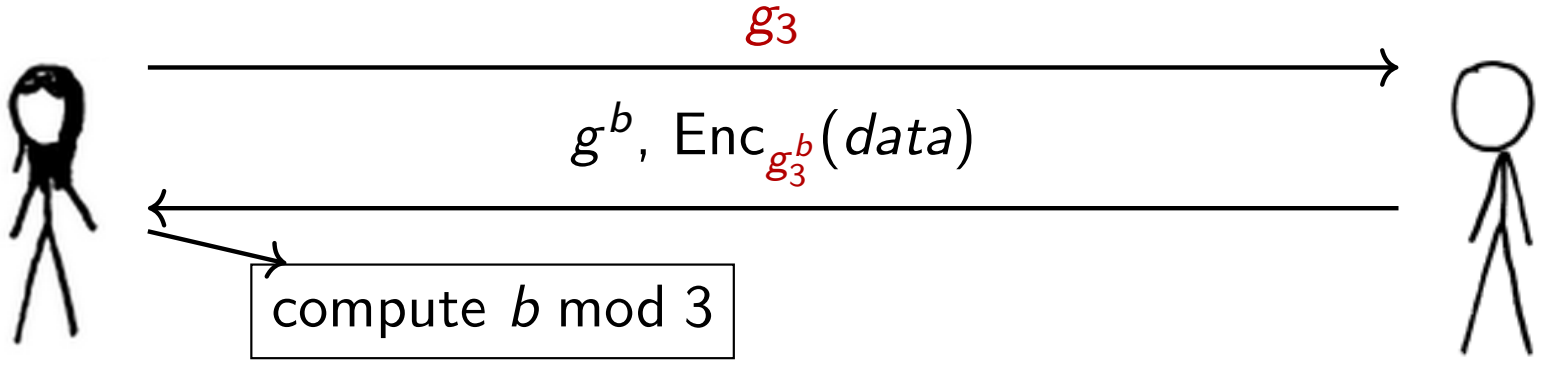

Repeat for many small subgroups $\Longrightarrow$ find b using Chinese Remainder Theorem

Small subgroup attacks

Made much worse with...

- Many small subgroups (i.e., p-1 has many small factors)
- Short secret exponents (common optimization)
- Reused Diffie-Hellman values (common optimization)

Countermeasures

The countermeasures against these attacks are well known, and built into every DH standard:

- Use a "safe" prime $p = 2q + 1$, where q is prime
  1. Verify $2 \leq y \leq p - 2$ (otherwise, may leak 1 bit)
- Use a subgroup of large prime order q mod p
  1. Verify $2 \leq y \leq p - 2$
  2. Verify $1 = y^q \bmod p$

Inspiration for our work

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Theorem (Murphy's law)

Anything that can go wrong, will go wrong.

Corollary

If it is possible for an implementation to have made a mistake, someone has.

Standards mandate smaller subgroups

Leaves room for implementation mistakes

NIST SP800-56a: Recommendation for Pair-Wise Key Establishment Schemes Using Discrete Logarithm Cryptography

Table 1: FFC parameter-size sets

FFC parameter-size set name	FA	FB	FC
Maximum security strength supported (in bits)	80	112	112
Bit length of field size p (i.e., $\lceil \log_2 p \rceil$)	1024	2048	2048[1]
Bit length of subgroup order q (i.e., $\lceil \log_2 q \rceil$)	160	224	256

- No extra benefit from using small subgroups when already using short exponents
- DSA needs small subgroups, but not DH

Fast internet scanning lets us study behavior of publicly accessible hosts.

Widely deployed RFC5114 groups follow NIST recommendations*:

Group			Host Counts			
Name	**p (bits)**	**q (bits)**	**HTTPS**	**SMTP**	**IKEv1**	**IKEv2**
Group 22	1024	160	3%	$\approx$ 0%	17%	13%
Group 23	2048	224	$\approx$ 0%	33%	17%	13%
Group 24	2048	256	$\approx$ 0%	$\approx$ 0%	18%	14%
Total	—	—	40.6M	3.4M	1.9M	1.3M

Group 23: Can recover **201** bits of exponent in $\approx 2^{42}$ work

*: Scans from November 2016

Hosts don't validate group order.

	DHE Hosts	Non-Safe Primes	Hosts accepting...
HTTPS	11M	14%	
IKEv1	2.6M	13%	
IKEv2	1.3M	14%	
SSH	11M	$\approx$ 0%	

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	DHE Hosts	Non-Safe Primes	Hosts accepting... 0
HTTPS	11M	14%	0.6%
IKEv1	2.6M	13%	*
IKEv2	1.3M	14%	*
SSH	11M	$\approx$ 0%	3%

*: Did not scan: 0 causes unpatched Libre/Openswan to restart IKE daemon.

Hosts don't validate group order.

	DHE Hosts	Non-Safe Primes	Hosts accepting... 0	1
HTTPS	11M	14%	0.6%	3%
IKEv1	2.6M	13%	*	28%
IKEv2	1.3M	14%	*	0%
SSH	11M	$\approx$ 0%	3%	25%

*: Did not scan: 0 causes unpatched Libre/Openswan to restart IKE daemon.

Hosts don't validate group order.

	DHE Hosts	**Non-Safe Primes**	**Hosts accepting...** 0	1	p-1
HTTPS	11M	14%	0.6%	3%	5%
IKEv1	2.6M	13%	*	28%	27%
IKEv2	1.3M	14%	*	0%	0%
SSH	11M	$\approx$ 0%	3%	25%	33%

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Hosts don't validate group order.

	DHE Hosts	**Non-Safe Primes**	**Hosts accepting...** 0	1	p-1	g_3/g_7
HTTPS	11M	14%	0.6%	3%	5%	$\approx 100\%$
IKEv1	2.6M	13%	*	28%	27%	99%
IKEv2	1.3M	14%	*	0%	0%	97%
SSH	11M	$\approx 0\%$	3%	25%	33%	N/A

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Libraries don't validate group order.

Similar findings to [DCE 2017 (up next!)]

Library (TLS)	**Validation**
Mozilla NSS	$g \leq 2$
OpenJDK	$g \leq 2$
OpenSSL 1.0.2	None*
BouncyCastle	$g \leq 2$
Cryptlib	$g \leq 2$
libTomCrypt	None
CryptoPP	None
Botan	None
GnuTLS	$g \leq 2$

*: before CVE-2016-0701 in Jan '16

- "The server obtains the DH parameters via a `PKCS#3` file which does not contain any subgroup information. This file format is the defacto standard across all crypto libraries."
- OpenSSL vulnerable to full Lim-Lee key recovery attack for RFC 5114 primes
- Amazon Load Balancer vulnerable to partial key recovery attack

Misconceptions

Academics

"There are many good reasons for using smaller subgroups, including **efficiency** and the fact that this setting matches the **theoretical security analyses of cryptosystems**."

Implementors

"safe primes (...) have quite some undesirable properties. They don't have a subgroup with size of the selected security parameter and that **requires them to use very large keys**."

Fact: Short exponents with safe primes and with small subgroups are both well-studied

Disconnects

Academics

"(...) it is only necessary to validate cryptographic parameters properly - **but this is very well-known**."

Implementors

"I bet there are TLS clients (and other DH users) out there that use those values, and we would break them (...) **functionality trumps security every day, and twice on Tuesdays**."

Countermeasures may be known, but are not always implemented

Takeaways

- Standards writers:
  - Software developers have different priorities
  - The fewer checks required, the better! (Murphy's Law)

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  - Project Wycheproof: test crypto libraries against known attacks (`https://github.com/google/wycheproof`)

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Questions?

References

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Kristen Dorey, Nicholas Chang-Fong, Aleksander Essex. *NDSS 2017*

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